

MOD-3 SUM \mathbb{Z}_3^4

A competitive game of skill for two or more players. Players find and collect sets of cards with certain overlapping patterns. The player with the largest collection of cards when all cards are played from the deck wins.

Game contents: 80 cards + instruction sheet

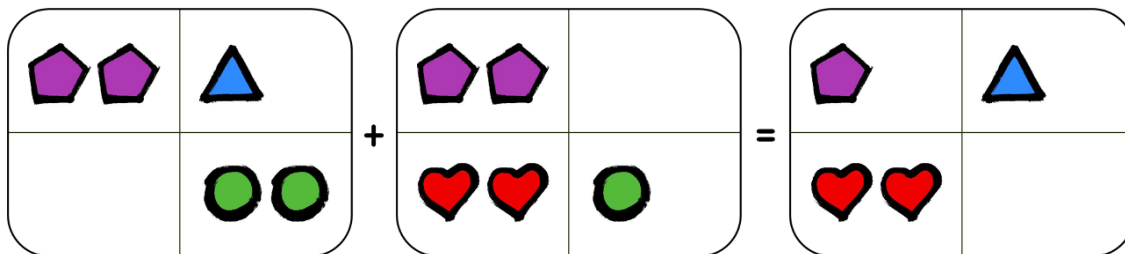


Object of the game

Five cards are placed face up on a table, each with the same horizontal orientation. The object of the game is to spot two different sets of cards that have the same "sum" and take them before someone else does. The cards that have been taken are then replaced by new cards and another hand is played. When the deck is finished, the player with the most cards wins.

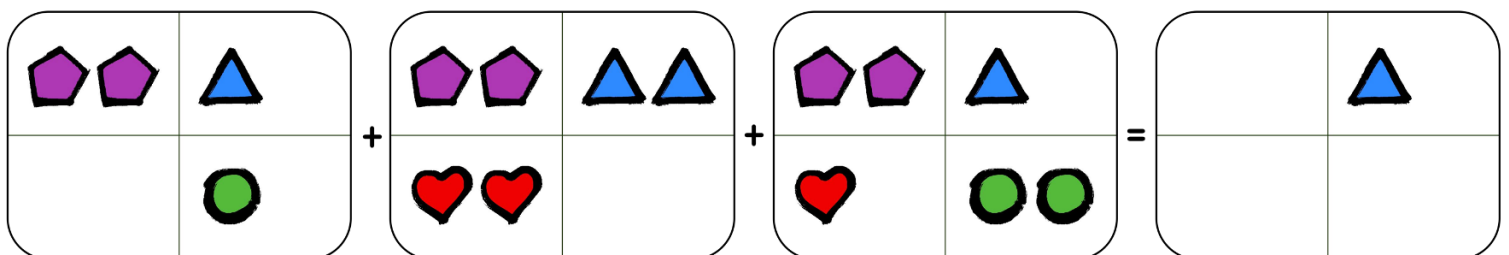
About the cards and summing them Mod-3

The cards are oriented horizontally, and each card is divided into four quadrants. The top-left quadrant of each card contains 0, 1 or 2 purple pentagons; the top-right quadrant contains 0, 1 or 2 blue triangles; the bottom-left quadrant contains 0, 1 or 2 red hearts, and finally, the bottom-right quadrant contains 0, 1 or 2 green circles. Think of the quadrants as slots for the respective shapes. Cards are summed together slot-wise, meaning that one adds the numbers of each shape in the respective quadrants with one another. This sum is then taken Mod-3, so that only the remainder after dividing by 3 matters, and the sum of the respective shapes is always 0, 1 or 2. A first example of this slot-wise addition of cards is given below:



We are summing the first two cards. We first sum the pentagons. 2 pentagons + 2 pentagons = 4 pentagons, but we are doing the summing Mod-3, so we just consider the remainder after dividing by 3, which is 1. Similarly, 1 triangle + 0 triangles = 1 triangle; 0 hearts + 2 hearts = 2 hearts, and 2 circles + 1 circle = 0 circles (Mod-3!). The card with the resulting 1 pentagon, 1 triangle, 2 hearts and 0 circles is shown on the right.

It is also possible to sum more than two cards together. An example of three cards being summed together is given below:

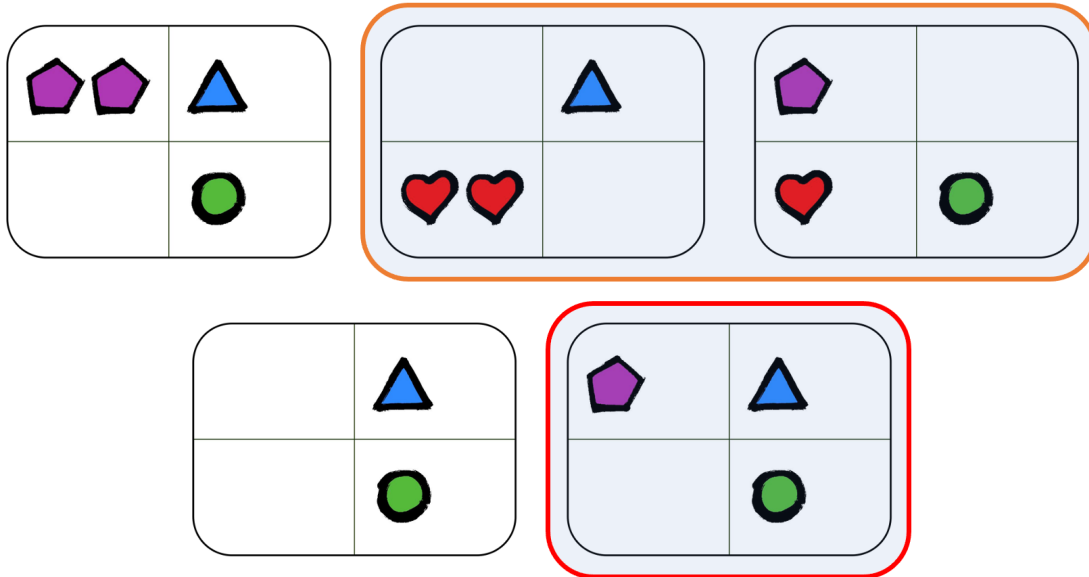


One can either sum the first two cards and add that sum to the 3rd card to get the final sum, or sum the respective shapes across all three cards, remembering to take the sum Mod-3, when filling the slots of the

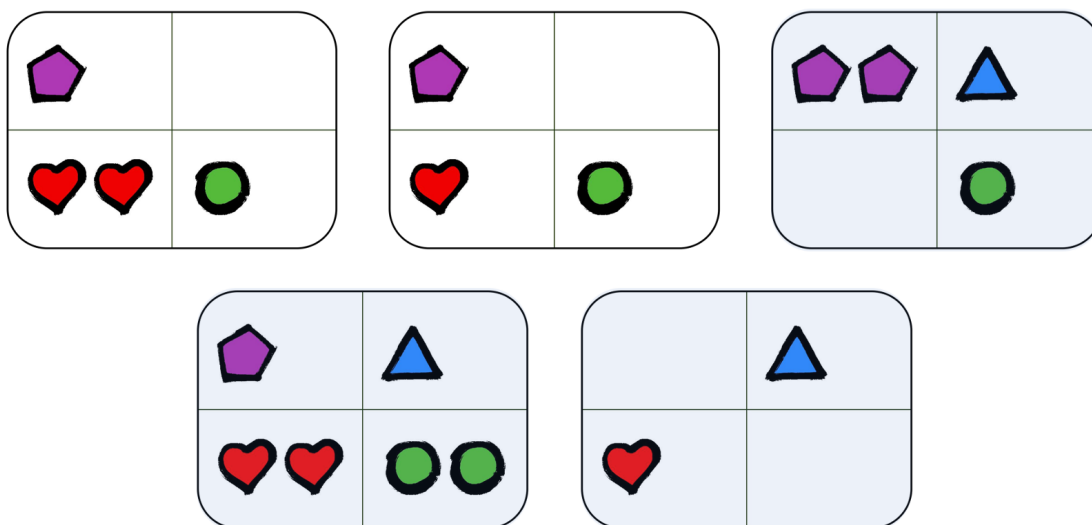
card representing the sum. In the above example, the sum of 2 pentagons + 2 pentagons + 2 pentagons = 6 pentagons = 0 pentagons (Mod-3). Hence the top-left quadrant of the card representing the sum is empty.

Finding subsets that have a common sum

The object of the game is to find two subsets of the dealt cards that have the same sum Mod-3. For example, in the hand depicted below,



the two highlighted cards in the top row sum to the same thing as the card in the bottom row. The player spotting these matching sums announces the find, and once verified by the other players, the player removes the cards from the game and adds them to their pile. It is also possible that some subset of the cards sums to an empty card. In this case, the second subset is the "empty subset" and the set of cards summing to the empty card may be taken in its own right. The set of cards below illustrates such a situation, with the highlighted cards in this case summing to the empty card, as you should be able to readily verify.



It is an interesting mathematical fact that given any collection of five cards there are always two subsets with the same Mod-3 sum.

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